$a + b\hat{i} + c\hat{q} + d\hat{o}$ or “echo” numbers

$$\hat{q}\hat{o} = 1, \hat{q}\hat{q} = \hat{i}, \hat{o}\hat{o} = -i, \hat{i}\hat{i} = -1, \hat{i}\hat{q} = -\hat{o}, \hat{i}\hat{o} = \hat{q}$$

addition

$$a + b\hat{i} + c\hat{q} + d\hat{o} = a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o} + a_2 + b_2\hat{i} + c_2\hat{q} + d_2\hat{o}$$

$$a = a_1 + a_2, b = b_1 + b_2, c = c_1 + c_2, d = d_1 + d_2$$

multiplication

$$a + b\hat{i} + c\hat{q} + d\hat{o} = (a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o})(a_2 + b_2\hat{i} + c_2\hat{q} + d_2\hat{o})$$

$$a = a_1a_2 - b_1b_2 + c_1d_2 + d_1c_2, b = a_1b_2 + b_1a_2 + c_1c_2 - d_1d_2, c = a_1c_2 + b_1d_2 + c_1a_2 + d_1b_2, d = a_1d_2 - b_1c_2 - c_1b_2 + d_1a_2$$

magnitude

$$|a + b\hat{i} + c\hat{q} + d\hat{o}| = \sqrt{\left(a^2 + b^2\right)^2 + \left(c^2 + d^2\right)^2 - 4(ad + cb)(ac - db)}$$

magnitude squared

$$||a + b\hat{i} + c\hat{q} + d\hat{o}|| = \left(a^2 + b^2\right)^2 + \left(c^2 + d^2\right)^2 - 4(ad + cb)(ac - db)$$

inversion

$$a + b\hat{i} + c\hat{q} + d\hat{o} = (a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o})^{-1}$$

$$a = \frac{a_1b_1^2 - b_1c_1^2 + b_1d_1^2 - 2a_1c_1d_1 + a_1^3}{||a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o}||}, b = \frac{a_1c_1^2 - a_1d_1^2 - a_1^2b_1 - 2b_1c_1d_1 - b_1^3}{||a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o}||}, c = \frac{-a_1^2c_1 + b_1^2c_1 + c_1^2d_1 + 2a_1b_1d_1 + d_1^3}{||a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o}||}, d = \frac{c_1d_1^2 - a_1^2d_1 + b_1^2d_1 - 2a_1b_1c_1 + c_1^3}{||a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o}||}$$

division

$$\frac{a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o}}{a_2 + b_2\hat{i} + c_2\hat{q} + d_2\hat{o}} = (a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o})(a_2 + b_2\hat{i} + c_2\hat{q} + d_2\hat{o})^{-1}$$

$a + b\hat{i} + c\hat{q} + d\hat{o}$ or “echo” numbers

conjugate

$$\overline{a + b\hat{i} + c\hat{q} + d\hat{o}} = \frac{||a + b\hat{i} + c\hat{q} + d\hat{o}||}{a + b\hat{i} + c\hat{q} + d\hat{o}}$$

dot product

$$(a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o}) \cdot (a_2 + b_2\hat{i} + c_2\hat{q} + d_2\hat{o}) = \sqrt{\frac{1}{2} \cdot |(a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o})| \cdot |(a_2 + b_2\hat{i} + c_2\hat{q} + d_2\hat{o})| \cdot (||(a_1 + b_1\hat{i} + c_1\hat{q} + d_1\hat{o})|| + ||(a_2 + b_2\hat{i} + c_2\hat{q} + d_2\hat{o})|| - ||((a_1 - a_2) + (b_1 - b_2)\hat{i} + (c_1 - c_2)\hat{q} + (c_1 - d_2)\hat{o})||)}$$

exponential

$$e^{a} = e^{a}$$

$$e^{b\hat{i}} = cos(b) + sin(b)\hat{i}$$

$$e^{c\hat{q}} = cos\left(\frac{c}{\sqrt{2}}\right)cosh\left(\frac{c}{\sqrt{2}}\right) + sin\left(\frac{c}{\sqrt{2}}\right)sinh\left(\frac{c}{\sqrt{2}}\right)\hat{i} + \left(e^{\frac{c}{\sqrt{2}}}sin\left(\frac{c}{\sqrt{2}} + \frac{\pi}{4}\right) - e^{\frac{-c}{\sqrt{2}}}cos\left(\frac{c}{\sqrt{2}} + \frac{\pi}{4}\right)\right)\frac{\hat{q}}{2} + \left(e^{\frac{c}{\sqrt{2}}}cos\left(\frac{c}{\sqrt{2}} + \frac{\pi}{4}\right) - e^{\frac{-c}{\sqrt{2}}}sin\left(\frac{c}{\sqrt{2}} + \frac{\pi}{4}\right)\right)\frac{\hat{o}}{2}$$

$$e^{d\hat{o}} = cos\left(\frac{d}{\sqrt{2}}\right)cosh\left(\frac{d}{\sqrt{2}}\right) - sin\left(\frac{d}{\sqrt{2}}\right)sinh\left(\frac{d}{\sqrt{2}}\right)\hat{i} + \left(e^{\frac{d}{\sqrt{2}}}cos\left(\frac{d}{\sqrt{2}} + \frac{\pi}{4}\right) - e^{\frac{-d}{\sqrt{2}}}sin\left(\frac{d}{\sqrt{2}} + \frac{\pi}{4}\right)\right)\frac{\hat{q}}{2} + \left(e^{\frac{d}{\sqrt{2}}}sin\left(\frac{d}{\sqrt{2}} + \frac{\pi}{4}\right) - e^{\frac{-d}{\sqrt{2}}}cos\left(\frac{d}{\sqrt{2}} + \frac{\pi}{4}\right)\right)\frac{\hat{o}}{2}$$

$$e^{a+b\hat{i}+c\hat{q}+d\hat{o}} = e^{a}e^{b\hat{i}}e^{c\hat{q}}e^{d\hat{o}}$$

logarithm (sloppy, but provides answer even with more dimensions for most cases, surly a better O[log(d)] answer could be made)

$$y_0 \approx ln(x) \approx n\left(e^{\frac{1}{n}} - 1 + \frac{1}{x} - \frac{e^{\frac{x}{n}}}{x} + (1-x)e^{\frac{x}{n}}\sum_{m=0}^{n-1}\frac{e^{\frac{-(x-1)m}{n^2}}}{(x-1)mn - xn^2}\right)\Bigg|^{n=\lfloor ||x||^{\frac{3}{4}} \rfloor}$$

$$y_{n+1} = y_n - \frac{e^{y_n} - x}{e^{y_n}}, \textit{ newtons method for successivly better approximations}$$

NOTE: these could be extended by dimensions of powers of two d1*d1=o, d2*d2=-o, d3*d3=q, d4*d4=-q, and likely any dimension